Course Descriptions

C&! 2024

As usual, we'll have a range of courses from *Accessible* courses that anyone at camp can enjoy to *Intermediate* courses that will be a bit more formal and where some mathematical maturity helps and also some *Advanced* courses that can take us well beyond calculus. If a course falls under two categories that means that it serves as both. These divisions are meant to help you know what to expect, but remember they are imprecise! Also remember, just because it is accessible doesn't mean it is easy! In every course we have the ability to take students as deeply into advanced mathematics as they want to go.

You'll find a string of letters next to each course name. Here's what they mean.

- **H** **Hands-on** course where you'll be making things and out of your seat doing things as part of learning
- **P** **Pencil and paper** work will be an important part of learning in this course
- **L** Course with significant **lecture** component
- **C** **Computer-based** course, laptop needed
- **E** **Exploratory** course where students are offered problems and staff support, but are expected to self-direct their work

<table>
<tr><th>Accessible</th><th>Intermediate</th><th>Advanced</th></tr>
<tr><td colspan="3">How Languages Work: Sound & Syntax (PL)</td></tr>
<tr><td colspan="3">How Languages Change: Indo-European Historical Linguistics (PL)</td></tr>
<tr><td>Art + Math (H)</td><td></td><td></td></tr>
<tr><td>Knot Theory (H)</td><td></td><td></td></tr>
<tr><td>Number Sense (HP)</td><td></td><td></td></tr>
<tr><td>Games and Numbers (P)</td><td></td><td></td></tr>
<tr><td>Logic and Arithmetic (PL)</td><td></td><td></td></tr>
<tr><td>Introduction to Algorithms (LC)</td><td></td><td></td></tr>
<tr><td colspan="2">Recurrences (P)</td><td></td></tr>
<tr><td colspan="2">Geometry of Curvature (HP)</td><td></td></tr>
<tr><td colspan="2">The Mathematics of Fairness (PE)</td><td></td></tr>
<tr><td></td><td>Topology (PL)</td><td></td></tr>
<tr><td></td><td>Interpolation (PL)</td><td></td></tr>
<tr><td></td><td>Combinatorics (PE)</td><td></td></tr>
<tr><td></td><td>Discrete Calculus (PL)</td><td></td></tr>
<tr><td></td><td>Infinitude of Primes (PL)</td><td></td></tr>
<tr><td></td><td>Probability and Evolution (HL)</td><td></td></tr>
<tr><td></td><td>It's a Small World After All! (PL)</td><td></td></tr>
<tr><td></td><td>Paradox and The Philosophical Computer (CL)</td><td></td></tr>
<tr><td></td><td colspan="2">How Languages Arise: Language Contact & Birth (PL)</td></tr>
<tr><td></td><td></td><td>Statistical Physics (PL)</td></tr>
<tr><td></td><td></td><td>Hyperbolic Geometry (PLC)</td></tr>
<tr><td></td><td></td><td>Geometry and Relativity (HPC)</td></tr>
<tr><td></td><td></td><td>The Prime Number Theorem (PL)</td></tr>
<tr><td></td><td></td><td>Randomness & Computation (PL)</td></tr>
<tr><td></td><td></td><td>Lambda Calculus and Undecidability (PL)</td></tr>
</table>

How Languages Work: Sound & Syntax

Todd Krause

Consider: you have some crazy thought. By some process you can't explain in detail, you pass this thought into a jumble of sounds. Those sounds smash into somebody's ear and — poof! — suddenly that thought's in their head too! How in the world does that work?!? The trick is, the sounds aren't such a jumble after all. There's method to the madness: there's a system, and somehow we all carry around a copy of this system in our heads. Are all those copies the same? How much can they differ and still work? What kinds of systems are there in the world? What do they have in common?

This class will provide an introduction to the basics of modern linguistic analysis. What are nouns? You've seen them in English, but do all languages have nouns? Do verbs work the same way in all languages? In this class we will learn to analyze the fundamental components of a language: the different types of words that a language has. We will also explore how we identify the rules by which the elements fit together. What is the system we all carry around in our heads? The real challenge facing you as a budding linguist: how can you figure all this out for a language you don't even know yet?!? You might be surprised how some of the thinking you use in mathematics will keep you in good stead when analyzing languages. So come explore one of the great mysteries of the human mind: how it creates language!

No particular linguistic knowledge assumed. But you'll want to be comfortable with logical thinking and banging your head against concepts and data that might not immediately seem to make sense. Some familiarity with the basic elements of English grammar — nouns, verbs, conjunctions … participles? … clauses? — will be helpful, though not required.

How Languages Change: Indo-European Historical Linguistics

Todd Krause

Do you speak English like Shakespeare? Do you even speak it like your parents or your teachers? Take some time to look at the differences and think about how those changes come about.

And did you know English is related to German? Look at some common words to check: *Vater* 'father', *Mutter* 'mother', *zwei* 'two', *Hand* 'hand'. But it's also related to Italian, Greek, Russian, and even Hindi. How does that work? What does it mean for languages to be "related"?

This class will look at how Historical Linguistics describes relationships among languages. We will mine English for clues as to how languages evolve over time. We will compare Old English to other ancient languages looking for similar patterns in both grammar and vocabulary: maybe Latin? Ancient Greek? Gothic? (Not just a way of dressing ….) Sanskrit? Old Church Slavonic? Tocharian? (Is that actually a thing? …) Get ready to climb the family tree and learn some new — old — languages!

Art + Math

Martin Strauss

In this course, we'll explore art history/criticism, math concepts, and art execution over topics that may include: symmetry, perspective and other projection, curvature, dimension (including fractals and dimension greater than three) and self reference.

Knot Theory

Vivian Fang

What are the mathematically different ways to tie your shoelaces? This will be a hands-on class where we learn about different types of knots and some common invariants to distinguish knots from each other.

Number Sense

Wendy Cho

Fractals, Pascal's Triangle, figurate numbers, games... where simple arithmetic can suddenly give rise to all sorts of fun. In this class we will explore what structures arithmetic can produce and how it happens. Once we see patterns and structure, we will start learning how to formalize our observations and move from conjecture to proof. Along the way, we discover strategies for decomposing complex problems to make them easier to solve.

Games and Numbers

David Metzler

How is a game like a number? How do you "add" two games to make a new one? How could you say that one game is "larger" or "smaller" than another? In this course we will explore some aspects of John Conway's theory of combinatorial games, and how they relate to his theory of surreal numbers. We will spend most of our time playing certain games that are designed to bring out the basic ideas of both theories, while always looking for patterns. We will see how numbers tell us about when games are won or lost, and how games tell us about the existence of new kinds of numbers, such as infinity minus one or one over infinity.

Logic and Arithmetic

Daniel Hader

At its core, mathematics is the study of structure, be it of numbers, symmetries, shapes, or whatever, so long as we can discuss something about it unambiguously. But how can we really be sure we're really talking about the same structures let alone talking about them unambiguously? In this course, we'll explore the topic of logic and use it to build up a really important structure in mathematics: the natural numbers. Along the way we'll learn about the various connectives that make up the language of logic ("and", "or", "implies", etc.), and we'll play with different types of "proof calculus" including natural deduction and sequent calculus. We'll also discuss what it means for a logical system to be sound or complete, and we'll investigate the axioms of arithmetic including the very important axiom of induction (or axioms depending on how the theory is formalized). Finally, by the end we'll have proved some fundamental results in arithmetic all the way from the ground up.

This class only requires an understanding of arithmetic, but students with a familiarity with logic and formal proofs will be able to dive a bit deeper.

Introduction to Algorithms

Julien Piet

This course offers an introduction to fundamental algorithms and data structures. Students will learn about essential algorithms such as sorting techniques, search algorithms, and basic data structures. The course will include collaborative activities in which we apply these algorithms to real-life situations. The course will end by introducing one of two more complex but useful algorithms: Dijkstra's shortest path algorithm, or the Gale-Shapley algorithm.

Recurrences

Matthew Cho

How many moves does it take to solve the Towers of Hanoi puzzle with 64 disks? How many regions can you get when you cut a circle with 100 straight lines? How many 30-bit bit strings are there with no neighboring "1"s? How do you pick a seat for the Josephus Problem? All of these problems have similar solutions: we can solve the problem by brute force for small numbers, then we can find a rule that lets us write a solution for a bigger number in terms of smaller ones. These rules are called recurrences and they are a powerful, general, and fundamental part of discrete mathematics.

In this class we will combine individual and collaborative problem solving to uncover general principles in the mathematics of recurrences.

Geometry of Curvature

Rolfe Schmidt

We'll start like a typical geometry class, with Euclid's Elements, Book 1. Once we get familiar with the postulates and using them to prove things, though, the real fun will begin. Instead of just proving harder theorems, we will get to the heart of the matter by breaking the rules and using paper models to build spaces where Euclid's postulates aren't true. Some will be disconnected, some will have edges, but the really pretty ones just can't be made flat. You'll see how other geometries are possible and how we can start understanding them. And you'll make some neat models. More advanced students will have the opportunity to dive deeper into Euclid and to explore connections between topology and curvature by proving a discrete analog of the Gauss-Bonnet Theorem.

The Mathematics of Fairness

Wendy Cho

Voting involves the "adding up" of votes. Representation is about "dividing" a small set of representatives among a larger group of constituents. Both of these "mathematical exercises" attempt to achieve fairness in some sense. In this course, we will explore the mathematics behind notions of fairness and discover that the mathematics of fairness is simultaneously simple as well as as complex as any mathematics you will encounter.

Topology

Jeremy Van Horn-Morris

Topology is one of the three classical branches of pure mathematics. You might have run across the phrase "a topologist is someone who can't tell the difference between a cup of coffee and a doughnut." Come join us to see why these are the same thing to a topologist! We'll discuss many strange topological spaces that will break our understanding and we'll develop the language required to analyze things topologically. No specific math background is required, but we will be working with some very abstract ideas, so some exposure to abstract mathematics is a plus. Come prepared *to attempt* to draw the movies you'll see in your mind and to develop tools to access the inaccessible!

Interpolation

Martin Strauss

Two points determine a line; three points determine a quadratic. That basic principle is at the heart of: the Fast Fourier Transform, to multiply $(ax + b)(cx + d)$ with less than four multiplications of coefficients (and multiply bigger polynomials quickly); error-correcting codes for communicating reliably over noisy channels; secret sharing, so that your secret can be recovered from any two of Google, Dropbox, and iCloud but no single service learns anything; and the uncertainty principle, that says we can't know both the time and frequency of any sound.

Combinatorics

Matthew Cho

We will cover different types of combinatorial techniques, such as identifying patterns, geometric probability, and recurrences. We will begin with simple problems and move toward more difficult problems with the idea that once you can grasp the basic tools and see them in more simple problems, you will learn to quickly identify good problem solving tactics for difficult problems.

Discrete Calculus

Daniel Hader

Calculus is often thought of as the study of infinitesimally small changes, but what about finitely small changes? Well, it turns out that many of the important results in calculus still hold true in this finite context with only a few tiny changes (pun intended). In this course we will explore the concept of the finite differences, an idea analogous to the derivative from calculus, but which does not require difficult concepts like "limits" to understand. We'll derive many of the important ideas from calculus including the chain and product rules as well as proving a discrete version of the fundamental theorem of calculus using only basic arithmetic and algebra. Additionally, we'll see how understanding finite differences helps simplify complicated tasks like finding a polynomial that goes through a desired set of points. By the end, we'll have learned the fundamental tenants of calculus without ever doing any actual calculus at all!

Students interested in taking this course should feel comfortable with polynomials, algebra, and graphing functions. No calculus knowledge is needed.

Infinitude of Primes

Yo'av Rieck

In this course we will prove that there are infinitely many primes. Starting with Euclid's classical proof, we will cover several proofs that give more and more intricate information about number of primes less than a given quantity and afford us a lot more information than just "infinitely many".

Probability and Evolution

James Degnan

We will look at how probability can be used to understand DNA sequences and how these can be used to determine which species are most closely related. Are humans closer to chimpanzees or to gorillas? If have DNA sequences, how many times would you see a certain word like GATTACA in an entire genome?

Prior knowledge of probability is not required - we'll review it from the ground up!

It's a Small World After All!

Tanay Mehta

As people have observed throughout the ages, one can connect two people through a short chain of acquaintances, a friend of a friend of a friend. This observation is often described as the 'small world phenomenon' or 'six degrees of separation'. In this course, we'll look at how to model this phenomenon using graph theory and look at some algorithms for finding other people in a social network based on who your friends are.

Interested students should be familiar with Algebra 2, basic probability and basic programming.

Paradox and The Philosophical Computer

James Degnan

We'll use computer programming and simulations in the R language to explore everything from recurrences and fractals to some seemingly paradoxical ideas such as the Prisoner's Dilemma, The Liar's Paradox, and the Monty Hall problem from probability.

Please bring a laptop. Background in R is not necessary. R is a free program that can be installed on Windows, Mac, or Linux.

How Languages Arise: Language Contact & Birth

Todd Krause

Languages unfortunately die all the time, but have you ever seen one being born? The traditional story about language birth is that it must've been some Tower of Babel event: there was some prehistoric time when everyone spoke one language, and then it divided over time into various languages. Maybe. But for the moment, that's still lost to prehistory ... shrouded in myth.

But some languages have been born in the historical period, right before our very eyes! They start out as pidgins and work their way to becoming creoles. And this has happened several times all across the globe. What are these languages like? How do they arise? How do they evolve?

In this class we'll study the origins of pidgin and creole languages. We'll investigate their structures and cultural settings. And we'll see how, even though they were born independently in completely different parts of the globe, they actually have more in common than you might expect. In fact, they might just tell us a lot about how human language works as a psychological system. Come explore pidgins and creoles with us: you'll find that you can actually already read some of the newest languages on Earth!

No particular prior knowledge of linguistics is assumed. But the classes How Languages Work and How Languages Change provide great background and motivation for studying the concepts we'll discuss here. Come with an open mind and an interest in finding novel systems in new settings.

Statistical Physics

Julien Piet

This course provides an introduction to the fundamental principles of statistical physics. Students will learn how to derive the macroscopic properties of systems from the microscopic behaviors of their individual components. Key topics include temperature and entropy. The course aims to provide a comprehensive understanding of how microscopic interactions give rise to observable phenomena.

A basic understanding of calculus is helpful. No background in physics is necessary, but knowing some basics can be helpful. Familiarity with basic probability concepts is also beneficial.

Hyperbolic Geometry

Jeremy Van Horn-Morris

Mathematicians spent over two thousand years trying to prove the redundancy of Euclid's parallel postulate. In the 1800s, several mathematicians (Gauss, Bolyai and Lobachevsky) discovered that there were alternative 2-dimensional geometries in which all of Euclid's postulates, save the fifth, still held. One of these is Hyperbolic geometry, an amazing universe that looks both similar and radically different to what we expect from 2 dimensions. We'll learn about the different models of hyperbolic space, how to understand circles, straight lines and triangles, the symmetries of hyperbolic space, and some of the strange things we would experience if we lived in a hyperbolic world. We'll explore by direct by hand calculation as well as using the computer to assist. We'll also develop some physical models to interact with to solidify our hyperbolic intuition.

In order to get the most out of the course, participants will be expected to know basic calculus (through u-substitution) and basic geometry.

Geometry and Relativity

David Metzler

You may have heard of "spacetime", but what is it? Well, it's where we all live! How should one think about it? In this course we will discover that it is best understood as an unusual kind of geometry, where certain rules are the reverse of what we expect. We will begin by making a careful analogy between ordinary Euclidean geometry and the geometry of spacetime in two dimensions, as described by Einstein's special relativity (interpreted mathematically by Hermann Minkowski). We will see that Newton's description of the world can be thought of as a kind of geometry on spacetime as well, one which is at least as strange from a geometrical point of view as Einstein's. We will focus particularly on some of the supposedly "paradoxical" results of special relativity and see how they are not paradoxes at all, just natural features of Minkowski geometry. If time allows we will pursue these ideas further and curve our geometry to understand gravity, the orbits of the planets, and perhaps the Big Bang and black holes.

Students should know basic Euclidean and analytic geometry (up through confidence with the distance *formula), how to graph an equation in the plane and be comfortable working with vectors and dot products.*

The Prime Number Theorem

Yo'av Rieck

This course is about the question: how many primes are there anyway? Of course, the basic answer is infinitely many, but can we do better? Can we estimate the number of primes less than 100? Or $1,000,000$? That is easy, we can use a computer to count them. But what about 10^{100}? Or 10^{1000}?

The Prime Number Theorem (PNT) discusses exactly this. The answer was conjectured by Legendre in 1797, and during the 19th century many leading mathematicians, including Gauss, Chebyshev, and Dirichlet worked on it. In 1859 Riemann published a short paper titled **Ueber die Anzahl der Primzahlen unter einer gegebenen Grösse** (on the number of primes less then a given magnitude). This short paper - $8\frac{1}{2}$ pages - became one of the most influantial papers in the history of math (it is the origin of, among many things, the Riemann Hypothesis).

In that paper Riemann introduced techniques of complex analysis to the study of prime numbers. Eventually, in 1898, de la Vallée Poussin and (independently) Hadamard used Riemann's ideas and proved PNT, showing that the number primes less than x is, assymptotically, $x/\log(x)$. Equivalently, the probability of a randomly chosen number x being prime is about $1/\log(x)$.

We will discuss complex analysis - calculus of functions from the complex plane to the complex plane - and find out many very surprising facts about them. We will then see how this leads to a proof of PNT.

Students should have a firm understanding of calculus.

Randomness & Computation

Tanay Mehta

We often think of computational power as requiring various *resources* such as time or memory; the more time or memory our algorithms have, the more we can accomplish. In this course, we'll focus on a different resource, *randomness*. Can random behavior improve the speed at which we solve problems? Can randomness make other problems harder for applications in cryptography? We'll explore different types of randomized algorithms, how to think about them, and see if randomness can help us improve the speed of *deterministic* algorithms.

Lambda Calculus and Undecidability

Vivian Fang

We'll show how any computation can be expressed by a function, and dig into what "undecidability" means. We'll cover concepts including the Church-Turing thesis, the halting problem, Gödel's incompleteness theorem, and Rice's theorem.