Campersand 2019: Course Descriptions

1 Overview

Below you will find a list of courses we are preparing for C! 2019. We've divided them into four types: Accessible, Intermediate, Advanced, and Research Track. You can read more about each course type in its section below. These divisions are meant to help you know what to expect, but remember they are imprecise! Also remember, just because it is accessible doesn't mean it is easy!

You'll find a string of letters next to each class name. Here's what they mean.

- (H) - Hands-on class where you'll be making things and out of your seat doing things as part of learning
- (P) - Pencil and paper work will be an important part of learning in this class
- (L) - Class with significant Lecture component
- (C) - Computer-based class, laptop needed
- (E) - Exploratory class where students are offered problems and staff support, but are expected to self-direct their work.
- (*) - Course with prerequisites. This course is challenging, if in doubt, talk to us about it.

Lower-case characters indicate that a feature is present in a class but not primary. A lower-case 'c' means that computers are useful but optional.

2 Accessible Courses

These classes will be accessible to all campers and do not require prerequisites. This does not mean they aren't interesting and challenging, though! While a couple of our courses, like Symmetries and Elliptic Curves, are offered at two levels, all of our courses are designed to go deep into their subjects. Our faculty will be ready to keep all students thinking and learning.

Computational Linguistics Problem Solving (P)
Instructor: Rolfe Schmidt

Can you decode a Hittite tablet? Or piece together the meaning of a Quechua phrase from a few known fragments? How on earth could people figure out conclusively that Linear-B was used to write a previously unknown dialect of ancient Greek?

In this class we will explore language problems from the North American Computational Linguistics Olympiad (NACLO). These problems often seem impossible to start. We will see that through systematic and careful reasoning - in other words with logic - we can unlock new knowledge with confidence that intuition will never provide.

Computer Organization (HLc)
Instructor: Rolfe Schmidt

Computers may seem complex but they are really just well organized systems of simple components. We'll start by building circuits to compute boolean functions and we'll see that these functions are really all we need. Not only that, but we'll discover how constructive mathematical induction can help build a circuit to compute any boolean function using copies of just one two-wire gate - a gate we can build with a transistor. With this in hand we will survey the von Neumann architecture at the heart of the modern computer and then take a peek at other ways we can make physics compute for us: with DNA, with billiard balls, and even with quantum mechanics.

Geometry Through Zometool (He)
Instructor: Paul Hildebrandt

Whenever a Zometool strut fits in a model, we can write and prove a theorem about it. By building and studying models thoughtfully we can learn geometry intuitively and formally at the same time. What is the diameter of an icosahedron? How many cubes can be inscribed in a dodecahedron? What shapes can be the shadow of a (hyper)cube? This class will run like a daily workshop where we balance creative building with guided discussion to build beautiful models and understand beautiful geometry.

Drawing (HP)
Instructor: Paul Hildebrandt

Learn how to draw! Start by learning to draw mathematical objects to develop your skills of careful observation and precision, then see where those skills can take you!

Exploring Mathematics (HP)

Instructor: Wendy K. Tam Cho

What seems like simple arithmetic can suddenly give rise to all sorts of fun. In this class we will put pencil (and pen and marker and scissors and tape) to paper and see first hand what structures arithmetic can produce and how it happens. Once we see patterns and structure, we will start learning how to formalize our observations and move from conjecture to proof.

Symmetries (PL)
Instructor: Ashley Ahlin

Symmetries show up in nearly every area of mathematics–number theory, geometry, combinatorics and others. Mathematical "groups" allow us to understand how symmetries apply across many different disciplines. The definition of a group is very simple, but gives us a surprisingly useful and beautiful tool. For example, we can understand exactly why, if you disassemble a Rubik's cube and reassemble it with just one cube twisted, then you will have no hope of solving the cube without taking it apart again. We will see how, starting with one magic square, you can generate a whole family of them. Groups will help us understand more about codes and ciphers, and see how check digits are used in credit card numbers and other important codes.

3 Intermediate Courses

Our intermediate courses also require few prerequisites, but they are a bit different. They require a bit more "mathematical maturity" or at least a strong enough interest to keep working hard in the face of perplexity.

Logic (PL)
Instructor: Chaim Goodman-Strauss

Logic lies at the foundation of mathematics and is full of wonderful paradoxes, brain-twisters and puzzles that you can use to amaze your friends and trick your parents. But more than that, Logic is the very language of mathematical thought and these tools will help you answer with confidence when faced with questions like "What is a theorem?" or "What is a proof?"

Rational Points on Elliptic Curves (LC)
Instructor: Burton Newman

Elliptic curves are just a small perturbation of what one learns in school: The curve defined by $x^2+y^2=1$ is a circle while the curve defined by $x^3+y^2=1$ is an elliptic curve. But the behavior of elliptic curves is much more interesting and unpredictable. Over the rational numbers, an elliptic curve may have a finite or infinite number of solutions, and deciding which is the case often requires sophisticated mathematical machinery. Furthermore, the set of solutions has additional structure: there is a way to 'add' two such solutions to obtain a third. This 'addition' rule satisfies many of the properties that integer addition does and is defined in a simple geometric way. The addition rule allows one to build new solutions from old, and to ask questions about the structure of the solution set that is not available for other sorts of curves. Elliptic curves played a key role in Andrew Wiles proof of Fermat's Last Theorem and Manjul Bhargava earned the 2014 Fields medal for making progress on their structure.

In this course we'll use computational tools like Magma and Sage to quickly formulate and test conjectures about rational points on elliptic curves. We'll then work together to try and explain what we are observing.

Prerequisites - Know how to solve quadratic equations and plot the solutions to equations like $x^2+y^2=1$ in the xy-plane. Some prior exposure to programming (e.g. 'if statements' and 'for loops').

Recurrences (PL)

Instructor: Tanay Mehta

How many moves does it take to solve the Towers of Hanoi puzzle with 64 disks? How many regions can you get when you cut a circle with 100 straight lines? How many 30-bit bit strings are there with no neighboring 1s? How do you pick a seat for the Josephus Problem? All of these problems have similar solutions: we can solve the problem by brute force for small numbers, then we can find a rule that lets us write a solution for a bigger number in terms of smaller ones. These rules are called recurrences and they are a powerful, general, and fundamental part of discrete mathematics.

In this class we will combine individual and collaborative problem solving to uncover general principles in the mathematics of recurrences.

Statistical Simulation (*CE)

Instructor: Wendy K. Tam Cho

Very few real-world problems have tidy, closed-form solutions. With numerical methods and approximation theorems in hand, you won't need to let this slow you down. In this class we will learn to use R Studio to model systems and analyze their statistical properties.

Prerequisites: Some prior exposure to programming (e.g. 'if statements' and 'for loops').

4 Advanced Courses

Our advanced courses require significant prerequisites and cover challenging material at a fast pace. If you are interested in one of these classes but worried about whether you are ready, reach out to us! We will help you decide and can help you make sure that you *are* ready.

All You Need Is Cubes (*HPLe)

Instructor: Paul Hildebrandt

In this class we will dive deep into the mathematics behind the Zome system and learn how the rich 2D and 3D models we build in the classroom (and outside the classroom!), like Penrose tilings, polyhedra, and honeycombs, are related to regular structures in 6, 10, even 15 dimensional spaces. Along the way we will see how a seemingly simple object, the (hyper)cubic lattice, and a simple operation, taking a lower dimensional 'shadow' of an object, are, in a sense, enough to give us 'everything'.

Prerequisites: Geometry

Rational Points on Elliptic Curves (Advanced) (*PLCe)

Instructor: Burton Newman

Note: This course will explore the same topics as its intermediate counterpart, but will move significantly faster.

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In this course we'll use computational tools like Magma and Sage to quickly formulate and test conjectures about rational points on elliptic curves. We'll then work together to try and explain what we are observing.

Prerequisites: Precalculus. Modular arithmetic. Some prior exposure to pro-

gramming (e.g. 'if statements' and 'for loops').

Quantum Computing (*PL)

Instructor: Tanay Mehta

Explore how quantum behavior gives us new computing primitives that appear to be fundamentally more powerful than our classical-physics based computers. This advanced class will teach you the basic elements of quantum computation. After starting with qubits, we will explore multi-qubit systems and entanglement, quantum teleportation, and quantum circuits. We will finish with a tour of some algorithms that form part of the anlage of today's field: Deutsch-Jozsa algorithm, Simon's algorithm, and Grover's search algorithm.

Prerequisites: Linear Algebra, probability theory, Boolean logic. Ask us, we can help you get this down before camp!

Symmetries, Groups, and Orbifolds (*HPc)

Instructor: Ashley Ahlin

Looking at a book of MC Escher artwork, or noticing patterns in tilework, the question naturally comes up of when two tessellations are the "same" in terms of their symmetries. Mathematical "groups" allow us to understand how symmetries apply to tessellations, as well as many other fields of mathematics. In this class, we will explore how we can use groups of symmetries to understand repeating patterns, including tessellations of the plane, regular polyhedra, and tesselations of hyperbolic space. We will introduce the idea of "orbifolds" to understand how symmetric patterns are created.

Prerequisites: Plane geometry

Undecidability (*PL)

Instructor: Chaim Goodman-Strauss

Will that program ever stop running? Can I keep putting these tiles together forever or will I get stuck? Is this the smallest circuit I can build that will compute a function? Is this theorem provable?

All good questions. All are *undecidable*. Take this class to learn what that means and learn how we can prove that there are some things we just can't know!

Prerequisites: Logic

5 Research Track

This is an experiment for 2019. We are going to look carefully at an area of current research interest and, if we get traction, try to keep the discussion going after camp is over.

Journal Club: The Discrete Logarithm Problem (*PE)
Instructor: Rolfe Schmidt

Explore the state of the art in modern cryptography by taking a deep dive into discrete-log and curve-based public key systems. We will use a recent review paper by Joux, Odlyzko, and Pierrot as our guide and learn not just about the power of these tools but also how to attack them.

This will not be a standard class. Participants will read and discuss material online before coming to camp and will meet outside of camp class hours to cover the material.

Students should either be enrolled in the *Elliptic Curves* course or be familiar with that material before camp.

Prerequisites: Elementary number theory, discrete math, elementary linear algebra, mathematical maturity

6 Recommendations

Some campers will look at this list and know exactly what they want to take. Others will not be sure. If you are in the latter group, here is a bit of guidance. If this isn't enough, reach out to us and ask!

6.1 First-time Campers

For first-time campers, especially those aged 11 and under, we recommend the Accessible courses. A good first-time camper program might look like this:

- Exploring Mathematics or Symmetries (or both!)
- Geometry Through Zometool
- Computer Organization or Computational Linguistics Problem Solving, if interested

In all of these courses we will make sure that the flow of the class is lively and campers have plenty of time up and out of their seats. Of course if an Intermediate course looks interesting you can mix that in too. Our *Logic* class would be a particularly good choice.

6.2 Returning Campers

Returning to C!? Well, you probably know the drill then. But please note that even though some of these classes look like repeats, they will all be different from last year. So if you enjoyed a class last year and you'd like a refresher, then it is fine to take it again.

6.3 Older Campers

For campers who might be new at C! but are old-hands at other math camps, look at our Intermediate and Advanced courses.

6.4 A Note About Placements

We will offer all campers a chance to rank their classes of interest, but unfortunately we cannot guarantee you'll get your top choices. For example, if two of your choices are scheduled at the same time in different rooms it might be difficult for you to fully participate in both. We will work hard to make sure all of our campers get good placements, and we may reach out to you during this process.